Econ 802

Final Exam

Greg Dow December 16, 2016

All questions have equal weight. If something is unclear, please ask.

1. Consider a firm with the production possibilities set Y. A typical production plan $y \in Y$ has some positive elements (outputs) and some negative elements (inputs).

(a) Assuming the profit function $\pi(p)$ is well defined, show that it is non-decreasing in the prices of the outputs (keep the prices of the inputs constant).

(b) Define constant returns to scale. Assuming CRS holds, under what conditions is the profit function is well defined? Explain using a graph for the two-good case.

(c) Assume there are one input and one output. You have observations on prices and quantities at two different points in time and the data are consistent with the weak axiom of profit maximization. Show the inner and outer sets YI and YO using a graph, explain your reasoning, and say why these sets are important.

2. Consider a firm with the production function $y = ax_1 + bx_2$ where $a > 0$ and $b > 0$. The inputs must satisfy $x_1 \geq 0$ and $x_2 \geq 0$. The input prices are $w = (w_1, w_2) > 0$.

(a) Construct a graph where the cost minimization problem has a boundary solution. Set up a Lagrangian with Kuhn-Tucker multipliers, write out <u>all</u> of the first order conditions, and show that the solution in your graph satisfies these conditions.

(b) The level of input 2 is fixed at $z_2 > 0$ in the short run. Describe the firm's short run supply function mathematically, show it on a graph, and explain it in words.

(c) Suppose the number of firms is fixed in the long run but both inputs are variable. Given a downward-sloping market demand curve, is there always an equilibrium price? Is the equilibrium price always unique? Explain using a graph.

3. Here are some miscellaneous questions about consumer theory.

(a) Assume the utility function $u(x)$ is strictly quasi-concave and differentiable. For a given consumption bundle $x^* > 0$, find the prices p^* such that the consumer wants x^* when income is $m = 1$. Briefly explain your reasoning.

(b) When the indirect utility function $v(p, m)$ is differentiable, $-\sum_i [\partial v(p,m)/\partial p_i]\, p_i = [\partial v(p,m)/\partial m]\, m$. What property of $v(p, m)$ leads to this result? Provide a verbal explanation and prove mathematically that the result is true.

(c) Suppose there are n consumers with indirect utility functions of the Gorman form $v_i(p, m_i) = a_i(p) + b(p)m_i$. Consider the aggregate consumer with utility $U = \sum u_i$ and income $M = \sum m_i$. If the price vector changes from p to p′, what new income level M′ does the aggregate consumer need in order to be exactly as well off as at (p, M)? Justify your reasoning.

4. Consider a pure exchange economy with two consumers (A and B) and two goods (1 and 2). A has utility $u_A(x_A) = -(1/x_{A1}) + x_{A2}$ and B has utility $u_B(x_B) = -(1/x_{B1}) + x_{B2}$. A is endowed with 1 unit of good 1 and zero of good 2. B is endowed with 4 units of good 2 and zero of good 1.

(a) Draw an Edgeworth box and label the origins and axes. Show the contract curve, justify your answer mathematically, and provide an economic interpretation.

(b) Compute an equilibrium price vector (p_1, p_2). Then draw another Edgeworth box showing (i) the endowment point; (ii) the budget line for each consumer; (iii) the equilibrium consumption bundles; and (iv) the indifference curve passing through the bundle for each consumer. Make sure your graph is numerically accurate.

(c) A social planner wants to choose the consumption bundles x_A and x_B to maximize the sum of the utilities $u_A + u_B$ subject to feasibility constraints on the total supply of each good. Does a solution exist? If so, is it unique? Explain your answers.

5. Robinson Crusoe is the only consumer. His utility function is $u(x, y) = \ln x + \ln y$ and he has an endowment $e > 0$ of the y good. He has a zero endowment of the x good. Acme is the only firm. It has the production function $x = \alpha z$ where z is the amount of the y good Acme uses as an input, and $\alpha > 0$ is a constant.

(a) Find the allocation that maximizes Crusoe's utility subject to the technology and endowment constraints (just use physical quantities; do not use any prices). Draw a graph with x on the horizontal axis and y on the vertical axis showing (i) the set of feasible allocations; (ii) the optimal allocation; (iii) the indifference curve that passes through Crusoe's consumption bundle; and (iv) the solution for z.

(b) Let p be the price of x and q be the price of y. Construct a Walrasian equilibrium that supports the allocation from (a). Prove that at the equilibrium prices, Crusoe is maximizing utility, Acme is maximizing profit, and both markets clear.

(c) There are n consumers with the same preferences and endowments as Crusoe, and there are m firms with the same technology as Acme. Assume that the consumers have ownership shares in firms $T_{ij} \geq 0$ with $\sum_i T_{ij} = 1$ for all j = 1 . . m. Prove that the price ratio from (b) gives a Walrasian equilibrium for this larger economy.